Problem Solving Lessons

Do I stand back and watch, or intervene and tell them what to do?

A PROFESSIONAL DEVELOPMENT MODULE

Introduction

The Formative Assessment Lessons are of two types; those that focus on the development of conceptual understanding and those that focus on problem solving. *Concept Development* lessons are intended to assess and develop students' understanding of fundamental concepts through activities that engage them in classifying and defining, representing concepts in multiple ways, testing and challenging common misconceptions, and exploring structure. *Problem Solving* lessons are intended to assess and develop students' capacity to *select* and deploy their mathematical knowledge in non-routine contexts and typically involve students in comparing and critiquing alternative *approaches* to solving a problem.

In this PD module, we focus on *Problem Solving* lessons. In most Mathematics classrooms, students are provided with structured tasks and are told precisely which techniques to use: students learn by following instructions. Even tasks described as 'problem solving' often amount to mathematical exercises rephrased in 'plain English', with a well-defined 'correct' solution method. Problems and situations that arise in the world, however, are rarely exercises in the use of a particular skill or concept. Such problems require students to make simplifications, model situations, *choose* appropriate knowledge and processes from their 'toolkit', and test whether their solution is 'good enough' for the purpose in hand. If students are to learn to use their skills autonomously in their future lives, they will need opportunities to work on less structured problems in their classrooms.

Each Problem Solving lesson begins with a less structured task and encourages students to formulate questions; choose appropriate representations and techniques; reason logically; construct hypotheses and arguments; compute accurately; interpret and evaluate results obtained; communicate the results and reflect on them. The teacher is then offered specific suggestions on how students' early attempts may be challenged and refined through collaborative activities. Students are given sample student work showing alternative approaches to the problem: these are usually incomplete or imperfect. They are asked to critique and improve these and then revise their initial attempts and/or try an alternative approach. The assessment of less structured problems has already been considered in the *Formative Assessment* module: rather than repeat that here, we explore the pedagogic demands of using such tasks in the classroom.

Activities

Acknowledgement:
In preparing this material, we acknowledge the permissions given by the Bowland Charitable Trust to adapt the professional development resources, *Bowland Maths*, that they had previously funded us to produce for the UK. This includes many of the handouts and most of the video extracts. Additional resources were also adapted from *Improving Learning in Mathematics*, a government funded program in the UK. The original sources are: Swan, M; Pead, D (2008). *Professional development resources*. Bowland Maths Key Stage 3, Bowland Trust/ Department for Children, Schools and Families. Obtainable in the UK from: http://www.bowlandmaths.org.uk. Swan, M; (2005). *Improving Learning in Mathematics*, challenges and strategies, Department for Education and Skills Standards Unit. Obtainable in the UK from http://tlp.excellencegateway.org.uk/pdf/Improving_learning_in_maths.pdf

Activity A: Revising structured problems

Time needed: 20 minutes.

Handout 1 presents three structured problems:

- Organizing a table tennis tournament
- Designing a box for 18 sweets
- Calculating Body Mass Index

These problems are of the same type as those found in many Mathematics classrooms. The first two are practical group tasks and the third is a computer-based task. The tasks are, however, structured so that they lead students through the problems, guiding and making decisions for them.

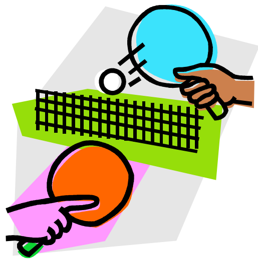
- Work through one of the structured problems carefully.
- List all the decisions that are being made for the students.
- Revise the problems so that some of these decisions are handed back to students. This will make them less structured.

For example, in *Organizing a table tennis tournament*, students are told:

- How to code the players (A, B, C…).
- To list all the matches that need to be played.
- How to systematically organize these matches.
- How to tabulate the order of play.
- To remember that players cannot play on two tables at once.

Handout 1: Structured problems

Organizing a table tennis tournament



You have the job of organizing a table tennis tournament.

- 7 players will take part
- All matches are singles.
- Every player has to play each of the other players once.

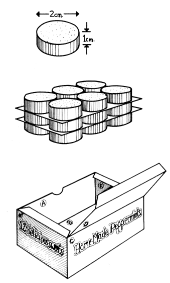
1. Call the players A, B, C, D, E, F, G
   Complete the list below to show all the matches that need to be played.

   A v B   B v C   ....
   A v C   B v D   ....
   ....

2. There are four tables at the club and each game takes half an hour.
   The first match will start at 1.00pm.

   Copy and complete the poster below to show the order of play, so that the tournament takes the shortest possible time.
   Remember that a player cannot be in two places at once!
   You may not need to use every row and column in the table.

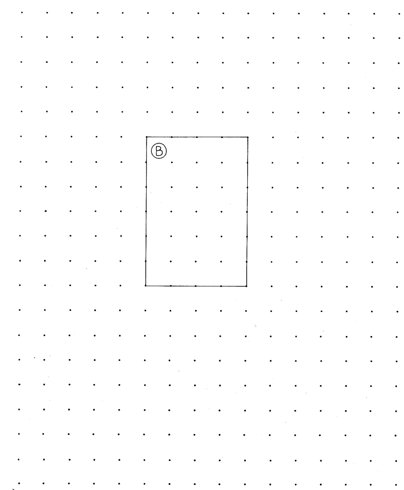
Designing a box for 18 sweets



You work for a design company and have been asked to design a box that will hold 18 mints.
Each mint is 2 cm in diameter and 1 cm thick.
The box must be made from a single sheet of card with as little cutting as possible.

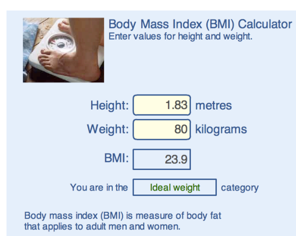
On the grid paper below, show clearly how the card can be folded up and glued together to make the box.

Make your box to check.



Calculating Body Mass Index

This calculator is used to help adults find out if they are overweight.



1. Fix the height at 2 meters - a very tall person!
   Complete the table below and draw a graph to show your results.

Weight (kg)	60	70	80	90	100	110	120	130	140
BMI									

   (a) What is the largest BMI for which someone is underweight?
   (b) What is the smallest BMI for which someone is overweight?
   (c) When you double the weight, what happens to the BMI?
   (d) Can you find a rule for calculating BMI from the weight?

2. Fix the weight at 80 kilograms and try varying the height.

   (a) When you double the height, what happens to the BMI?
   (b) Can you find a rule for calculating BMI from the height?
   (c) Draw a graph to show the relationship between the height and the BMI.

Note for students: If you put your own details into this calculator, *don't take the results too seriously!* It is designed for adults *who have stopped growing* and will give misleading results for children or teenagers!

Activity B: Compare structured and unstructured problems

Time needed: 10 minutes.

Handout 2 contains unstructured versions of the same tasks that were used in **Activity A**.

- Compare the less structured versions of the problems with the structured versions.
- What decisions have been left to the students?
- What pedagogical issues will arise when you start to use unstructured problems like this?

Some immediate issues that teachers raise are:

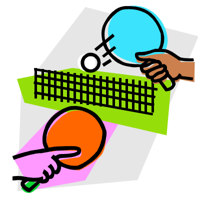
- Unstructured problems are more difficult.
- It is more difficult to plan a lesson with these problems.
- Students may not even know how to get started on them. Will we therefore need to structure them anyway?
- Students will not necessarily use what we have taught them.
- If we offer help too quickly, students will simply do what we say and not think for themselves.
- Students will generate a greater variety of approaches and solutions.
- Students may need reassurance that it is OK to try a different approach or reach a different conclusion.

Handout 3 contains some notes on the solutions to the three problems.

Handout 2: Unstructured versions of the problems

Organizing a table tennis tournament

You have the job of organizing a table tennis league.

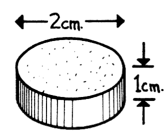


- 7 players will take part.
- All matches are singles.
- Every player has to play each of the other players once.
- There are four tables at the club.
- Games will take up to half an hour.
- The first match will start at 1.00pm.

Plan how to organize the league, so that the tournament will take the shortest possible time. Put all the information on a poster so that the players can easily understand what to do.

Designing a box for 18 sweets

You work for a design company and have been asked to design a box that will hold 18 sweets.
Each sweet is 2 cm in diameter and 1 cm thick.
The box must be made from a single sheet of card with as little cutting as possible.



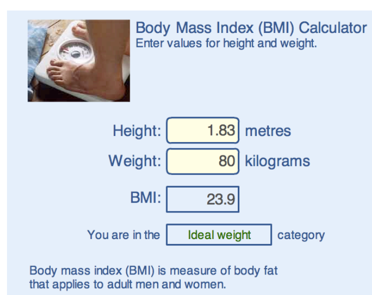
Compare two possible designs for the box and say which is best and why.

Make your box.

Calculating Body Mass Index

This calculator shown is used on websites to help an adult decide if he or she is overweight.
What values of the BMI indicate whether an adult is underweight, overweight, obese, or very obese?

Investigate how the calculator works out the BMI from the height and weight.



Note for students: If you put your own details into this calculator, *don't take the results too seriously!* It is designed for adults *who have stopped growing* and will give misleading results for children or teenagers!

Activity C: Consider strategies for offering help

Time needed: 15 minutes.

Teachers often find it difficult to know when to give help and when to leave students struggling. If they intervene too quickly, then the students have no chance to experience what it is like to pursue an unfruitful idea, or to puzzle out a solution for themselves. If they intervene too slowly, then students become frustrated, bored and disengaged.

Handout 4 contains some practical advice when using unstructured problems. Consider this advice carefully:

- Which ideas do you normally find most difficult to implement? Why is this?
- Is there any other advice you would add to this list? Write your own ideas at the bottom.

Bruner uses the metaphor of *scaffolding* to describe the structuring that a teacher provides (D. Wood, Bruner, & Ross, 1976)[1]. The teacher encourages students to do as much as they are capable of unaided and only provides the *minimum* of support to help them succeed. This support may involve reducing their choices, drawing attention to important features through questioning, or even at times demonstrating what to do. In his work with young children, Wood (1988)[2] categorized different levels of scaffolding, from less directive to more directive: giving general verbal advice, giving specific verbal instructions, breaking the problem down, demonstrating a solution. Wood also introduced two rules of contingency:

> *Any failure by a child to succeed in an action after a given level of help should be met by an immediate increase in help or control. Success by a child then indicates that any subsequent instruction should offer less help than that which preceded the success, to allow the child to develop independence. Wood (1988)*

The important idea here is that scaffolding should be removed as the student begins to cope otherwise it reinforces dependency.

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[1] Wood, D., Bruner, J., & Ross, G. (1976) The role of tutoring in problem solving. *Journal of child psychology and psychiatry*, 17, 89-100.

[2] Wood, D. (1988) *How Children Think and Learn*. Oxford and Cambridge, MA: Blackwell.

Handout 4: Practical advice for teaching problem solving

Allow students time to understand and engage with the problem Discourage students from rushing in too quickly or from asking you to help too soon.	• *Take you time, don't rush.* • *What do you know?* • *What are you trying to do?* • *What is fixed? What can be changed?* • *Don't ask for help too quickly - try to think it out between you.*
Offer strategic rather than technical hints Avoid simplifying problems for students by breaking it down into steps.	• *How could you get started on this problem?* • *What have you tried so far?* • *Can you try a specific example?* • *How can you be systematic here?* • *Can you think of a helpful representation?*
Encourage students to consider alternative methods and approaches Encourage students to compare their own methods.	• *Is there another way of doing this?* • *Describe your method to the rest of the group* • *Which of these two methods do you prefer and why?*
Encourage explanation Make students do the reasoning, and encourage them to explain to one another.	• *Can you explain your method?* • *Can you explain that again differently?* • *Can you put what Sarah just said into your own words?* • *Can you write that down?*
Model thinking and powerful methods When students have done all they can, they will learn from being shown a powerful, elegant approach. If this is done at the beginning, however, they will simply imitate the method and not appreciate why it was needed.	• *Now I'm going to try this problem myself, thinking aloud.* • *I might make some mistakes here - try to spot them for me.* • *This is one way of improving the solution.*

Activity D: Observe and analyze a lesson

Time needed: 30 minutes.

As we have seen, students (and adults!) do not always discuss in helpful ways. Some are reluctant to talk at all, while others take over and dominate. Students therefore need to be taught *how* to discuss. Some teachers have found it helpful to introduce a list of 'ground rules for discussion' into their classes. These ground rules should, in appropriate language, give explicit guidance to students on how to talk together profitably.

The two **Activity D Videos** show students working with the unstructured versions of the same problems you have worked on. Watch Michelle using the 'Organizing a table tennis tournament' problem.
As you watch the video, consider:

- How did the teacher organize the lesson? What phases did it go through?
- What resources did the teacher have available, and when were these used?
- Why were students expected to work in pairs/small groups?
- How did the teacher introduce the problem to students?
- What different approaches did students use?
- How did the teacher support the students that were struggling?
- How did the teacher encourage the sharing of approaches and strategies?
- What do you think these students were learning?

Afterwards you may enjoy watching the second video clip of Judith's lesson where she asks the class to design a box to hold 18 sweets.

Michelle begins the lesson by introducing the problem and also by explaining how the students should work together. The students are then given 3-4 minutes to write their individual, initial ideas of how they will tackle the problem. This helps them to formulate their ideas and have some ideas to contribute to the group discussions. Students were then given a few minutes to discuss their ideas. The video shows students getting to grips with the problem and the constraints involved. Some for example realize that they will not need all 4 tables.

At this point, Michelle introduces the resources that are available for them to use. She emphasizes that they should not rush the problem solving and that some may not get finished. She says that the important thing is to think about the approaches.

While groups work, Michelle first listens, then intervenes to promote deeper thinking:

> *"You've found one approach. You've found that it is not going to quite work, is it? So what do we need to do from there?"*
>
> *"Re-read the problem. Look at the last two sentences."*

Students use a wide range of representations and resources to solve the problem. Some used tables, while others used counters. These methods are shared in the final whole-class discussion.

Activity E: Plan a lesson, teach it and reflect on the outcomes

Time needed:
15 minutes discussion before the lesson.
1 hour for the lesson.
15 minutes after the lesson.

Choose one of the three problems that you feel would be appropriate for your class.

Discuss how you will:

- Organize the classroom and the resources needed.
- Introduce the problem to students.
- Explain to students how you want them to work together.
- Challenge/assist students that find the problem straightforward/difficult.
- Help them share and learn from alternative problem-solving strategies.
- Conclude the lesson.

If you are working on this module with a group, it will be helpful if each participant chooses the same problem, as this will facilitate the follow-up discussion.

Now you have taught the lesson, it is time to reflect on what happened.

- What range of responses did students have to this way of working?
  Did some appear confident? Did some need help? What sort of help? Why did they need it?
- What support and guidance did you feel obliged to give?
  Why was this? Did you give too much or too little help?
- What different strategies did students use?
  Share two or three different examples of students' work.
- What do you think students learned from this lesson?

If there is time, you may also like to watch the **Activity E Videos**. These show the teachers as they reflect on their own lessons with the table tennis and sweet box problems.

Suggested further reading

The seminal text for asking students to think mathematically:

Mason, J., Burton, L. and Stacey, K. (1982) *Thinking Mathematically,* London: Addison-Wesley

The book that inspired so much of the research into problem solving heuristics (or what to do when you are 'stuck'):

Polya, G. (1957) *How to Solve It: A New Aspect of Mathematical Method*, (2nd Ed) Penguin Science.

References

Wood, D. (1988) *How Children Think and Learn*. Oxford and Cambridge, MA: Blackwell.

Wood, D., Bruner, J., & Ross, G. (1976) The role of tutoring in problem solving. *Journal of child psychology and psychiatry*, 17, 89-100.

MAP Lessons for Formative Assessment of Problem Solving

The MAP lessons for formative assessment of problem solving produced, to date, are:

High School

Optimization Problems: Boomerangs *
Generalizing Patterns: Table Tiles *
Interpreting Statistics: A Case of Muddying the Waters *
Modeling With Geometry: Rolling Cups
Geometry Problems: Circles and Triangles *
Inscribing and circumscribing right triangles *
Solving geometry problems: Floodlights
Proofs of the Pythagorean Theorem
Modeling Conditional Probabilities 1: Lucky Dip *
Medical Testing
Modeling: Having Kittens *
Devising a Measure for Correlation
Solving Quadratic Equations: Cutting Corners

Middle School

Estimating and Approximating: The Money Munchers *
Estimating: Counting Trees
Design a Sports Bag
Maximizing Area: Golden Rectangles
Developing a Sense of Scale
Baseball Jerseys
Modeling: Making Matchsticks *

Lessons marked * are available to all from the site map.mathshell.org. The other lessons will be released in Q2 2012, and are available in preview form, on request, to Mathematics Development Collaborative partners.