

Formative Assessment

How can I respond to students in ways that improve their learning?

HANDOUTS FOR TEACHERS

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Handout 1: Difficulties in formative assessment

The research literature suggests that formative assessment practices are beset with problems and difficulties. These are summarized in the extensive review by Black and Wiliam (1998)¹ as follows:

Effectiveness of learning:

- Teachers' tests encourage rote and superficial learning.
- The questions and methods used are not shared between teachers, and they are not critically reviewed in relation to what they actually assess.
- There is a tendency to emphasize quantity of work and to neglect its quality in relation to learning.

Impact of assessment

- The giving of scores and the grading function are overemphasized, while the giving of useful advice and the learning function are underemphasized.
- Approaches are used in which students are compared with one another, the prime purpose of which seems to them to be competition rather than personal improvement; in consequence, assessment feedback teaches low-achieving students that they lack "ability," causing them to come to believe that they are not able to learn.

Managerial role of assessment

- Teachers' feedback to students seems to serve social and managerial functions, often at the expense of the learning function.
- Teachers are often able to predict students' results on external tests because their own tests imitate them, but at the same time teachers know too little about their learning needs.
- The collection of marks to fill in records is given higher priority than the analysis of students' work to discern learning needs; furthermore, some teachers pay no attention to the assessment records of their students' previous teachers.

¹ Black, P., & Wiliam, D. (1998). *Inside the black box: raising standards through classroom assessment*. London: King's College London School of Education.

Handout 2: Principles for formative assessment

Formative assessment may be defined as:

... all those activities undertaken by teachers, and by their students in assessing themselves, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged. Such assessment becomes 'formative assessment' when the evidence is actually used to adapt the teaching work to meet the needs. (Black & William, 1998)

Make the objectives of the lesson explicit

Share the objectives with students and from time to time ask students to produce evidence that they can achieve these objectives.

*"Make up an example to show me that you know and understand Pythagoras' theorem."
"This lesson was about you deciding what methods to use. Show me where you did this."*

Students may find it difficult to appreciate that some lessons are concerned with understanding concepts, while others are more concerned with developing mathematical practices. Making objectives explicit doesn't mean writing them on the board at the beginning of the lesson, but rather referring to them explicitly while students are working. If the objectives are to develop mathematical practices then in plenary sessions, ask students to share and compare approaches, rather than answers.

Assess groups as well as individual students

Group activities allow many opportunities to observe, listen, and question students. They help to externalize reasoning and allow the teacher to see quickly where difficulties have arisen.

Watch and listen before intervening

Before intervening in a group discussion, wait and listen. Try to follow the line of reasoning that students are taking. When you do intervene, begin by asking them to explain something. If they are unsuccessful then ask another student to help.

Use divergent assessment methods ("Show me what you know about ...").

Convergent assessment strategies are characterized by tick lists and can-do statements. The teacher asks closed questions in order to ascertain whether or not the student knows, understands or can do a predetermined thing. This is the type of assessment most used in written tests.

Divergent assessment, in contrast, involves asking open questions that allow students opportunities to describe and explain their thinking and reasoning. These questions allow students to surprise us - the outcome is not predetermined.

Give constructive, useful feedback

Research shows that responding to students' work with marks or levels is ineffective and may even obstruct learning. Quantitative feedback of this type results in students comparing marks or levels and detracts from the mathematics itself.

Instead, use qualitative oral and written comments that help students recognize what they can do, what they need to be able to do and how they might narrow the gap.

Change teaching to take account of assessment

As well as providing feedback to students, good assessment feeds forward into teaching. Be flexible and prepared to change your teaching plans in mid-course as a result of what you discover.

Adapted from: Improving Learning in Mathematics, Department for Education and Skills, 2005.

Handout 3: Making reasoning visible

Use questioning with mini-whiteboards

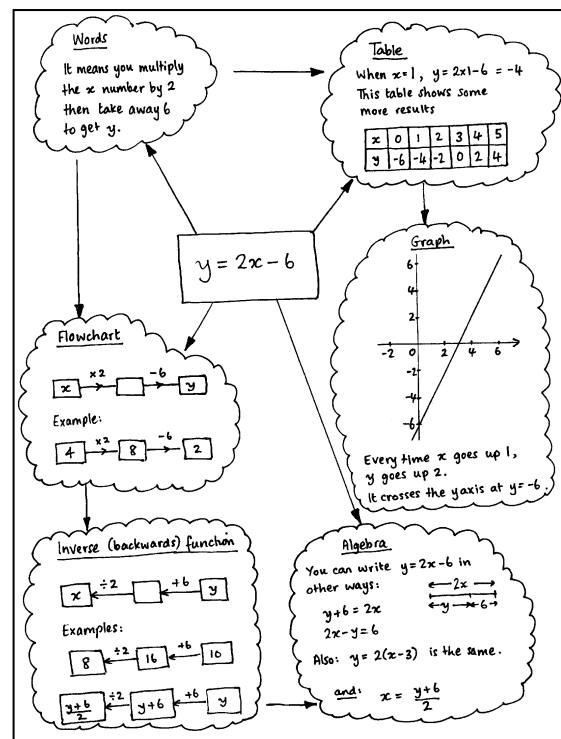
One difficulty with normal classroom questioning is that some students dominate while others are too afraid to participate. In this strategy, every student presents a response simultaneously. When open questions are used, students are able to give different responses to those around them. The teacher is able to immediately assess which students understand the ideas and which are struggling.



Ask students to produce posters

Ask each small group of students to work together to produce a poster:

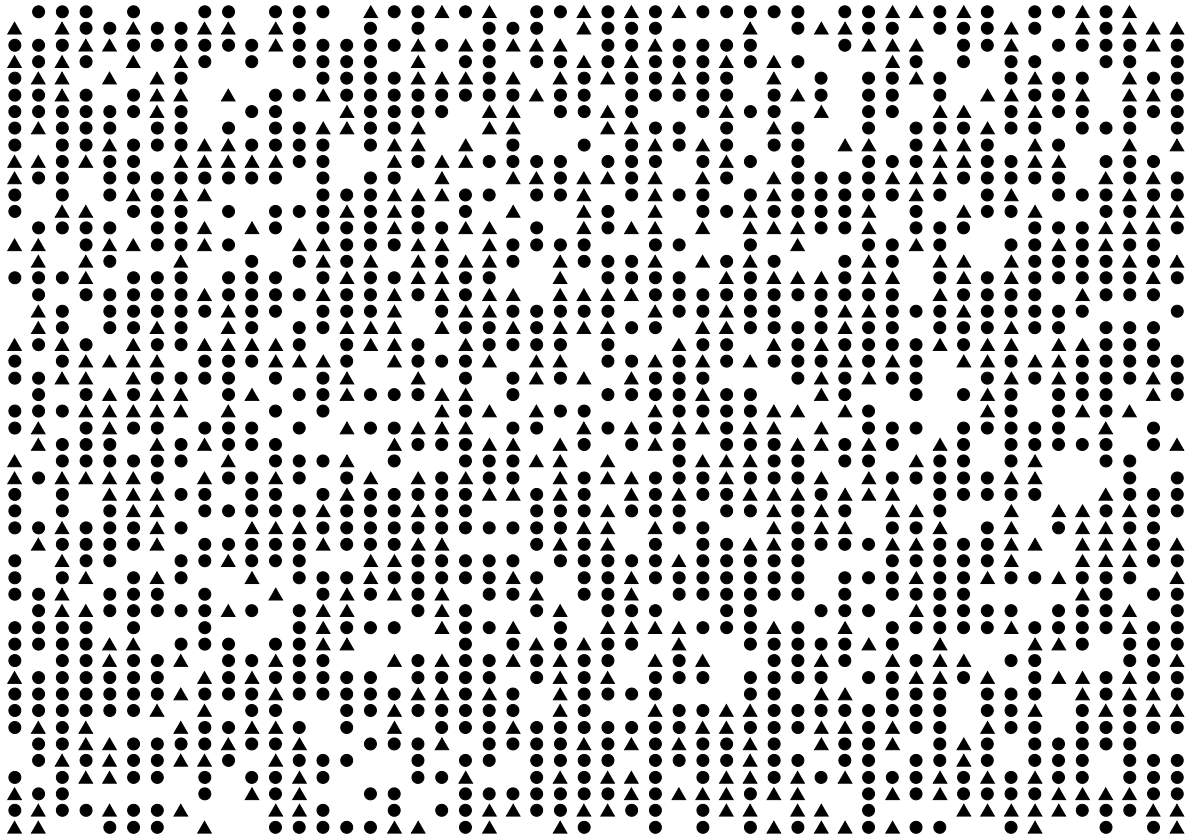
- showing their joint solution to a problem
- summarizing what they know about a given topic, or
- showing two different ways to solve a given problem
- showing the connections between a mathematical idea and other related ideas.



Adapted from: *Improving Learning in Mathematics*, Department for Education and Skills, 2005.

Handout 4: Assessment tasks and sample responses

Counting Trees



This diagram shows some trees in a tree farm.

The circles ● show old trees and the triangles ▲ show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

1. What method could he use to estimate the number of trees of each type?
Explain your method fully.
2. On your worksheet, use your method to estimate the number of:
 - (a) Old trees
 - (b) Young trees

Sample response: Laura

① You could multiply the number of trees in the length by the number of trees in the width and then half your answer.

② a. Old trees - 644
Young trees - 644

width - 33 $33 \times 39 = 1287$
length - 39. $1287 \div 2 = 643.5 = 644$

Sample response: Jenny

1. there are 38 trees in each column
there are around 11 young trees
and around 27 old ones
33 trees in each row so

$$11 \times 33 = 363$$

$$27 \times 33 = \underline{891}$$

$$\underline{1254}$$

2.

a. $11 \times 33 = 363 = \text{new trees.}$

b. $27 \times 33 = 891 = \text{old trees.}$

Sample response: Woody

2 columns has 21 young trees
55 old trees

50 columns is approx
 $50 \div 2 = 25$
 $25 \times 21 =$ amount of young trees $= 525$
 $25 \times 55 =$ amount of old trees $= 1,375$
 rounded up
 young 530
 old 1,380

Sample response: Amber

Counting trees

- If Tom draws a 10x10 square round some trees and counts how many old and new there are. There are 50 rows and 50 columns altogether so he must multiply by 25. He could do this a few times to check and then take the average.
- | | | | |
|------------|---------------|-------------|--|
| 53 old | $\times 25 =$ | 1325 old | |
| 28 new | $\times 25 =$ | 700 new | |
| 19 spaces | $\times 25 =$ | 475 spaces | |
| <u>100</u> | | <u>2500</u> | |

$1325 + 1200 \div 2 = 1262.5$
 $700 + 875 \div 2 = 787.5$

check

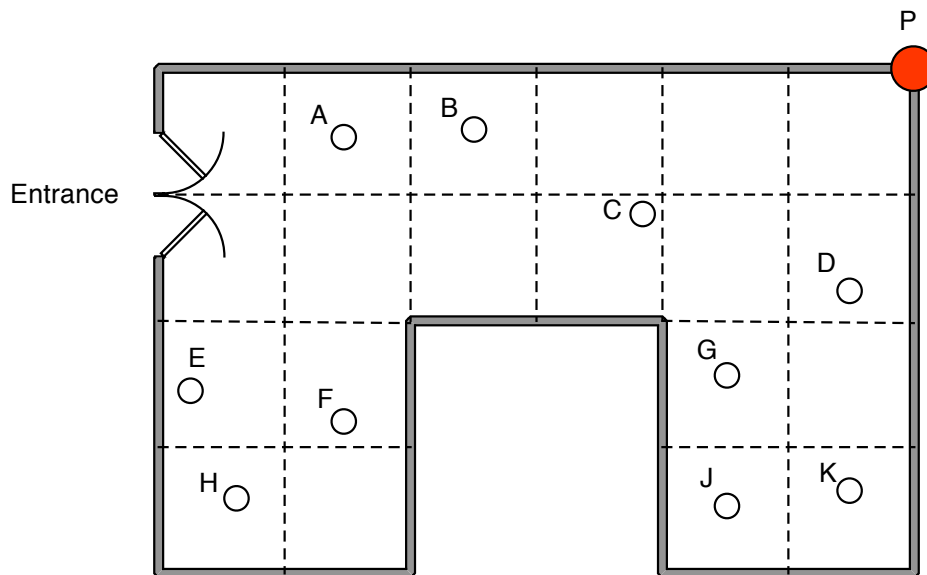
<u>48</u> old	$\times 25 =$	1200 old	So about 1263 old trees and 788 new trees
35 new	$\times 25 =$	875 new	
<u>17</u> spaces	$\times 25 =$	<u>425</u> spaces	
<u>100</u>		<u>2500</u>	

Handout 4: Assessment tasks and sample responses (continued)

Security Camera

A shop owner wants to prevent shoplifting.
 He decides to install a security camera on the ceiling of his shop.
 The camera can turn right round through 360° .
 The shop owner places the camera at point P, in the corner of the shop.
 The plan below shows ten people are standing in the shop.

Plan view of the shop



- Which people cannot be seen by the camera at P?
- The shopkeeper says that "15% of the shop is hidden from the camera"
Show clearly that he is right.
- (a) Show the best place for the camera, so that it can see as much of the shop as possible.
 (b) Explain how you know that this is the best place for the camera.

Sample response: Max

1. E, F and H cannot be seen by the camera.
 2.
 3a. The exact middle of the shop would be the place where it could see the most amount of people.
 3b. Because the middle of the shop will grant the camera a larger vision of the shop.

Sample response: Ellie

1. F + H

2. This is true because if there are 20 squared areas to make up the shop and 3 cannot be seen by the camera then that means the 3 squared areas would have to equal 15%. They do because if ^{15%} of the room = 100% then to get from 10 to 100 you divide by 10 and if you get 5 to 100 you divide by 2 and then by 10. add them together and you'll get 15%.

3a/b. I think the best place for the camera is in the centre of the room because it only can't see two squares.

Sample response: Simon

1. F+H

2. because 3 squares a hidden from the camera
 1 square is 50% so 3 squares are 150%

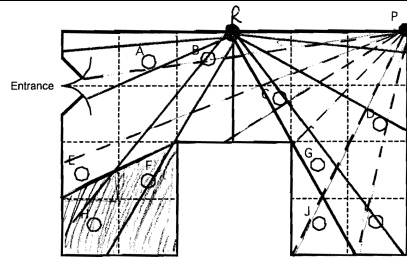
3.

a. Here is the best place

b. it can see all the cars almost everywhere

The diagram shows a 3x5 grid of squares. A camera is positioned on the left edge, and a car is on the right edge. A path of sight is drawn from the camera through squares A, B, F, and G to the car. Other squares are labeled C, D, E, H, I, J, and K. The path is shaded to indicate visibility.

Sample response: Rhianna



1. He cannot see F + H.
2. There are 20 squares. 3 squares are hidden from the camera.
Each square represents 5%.
 $3 \times 5\% = 15\%$
This proves 15% of the shop is hidden

3.

- a) ● = R
My camera 5% is hidden on one half.
5% is hidden on the other half.

This way only 10% is hidden + that space could be used for a til/trolleys.

- b) I know this is the best place because it has a full view of all around the shop it can go .

Handout 4: Assessment tasks and sample responses (continued)

Cats and kittens

Here is a poster published by an organization that looks after stray cats.



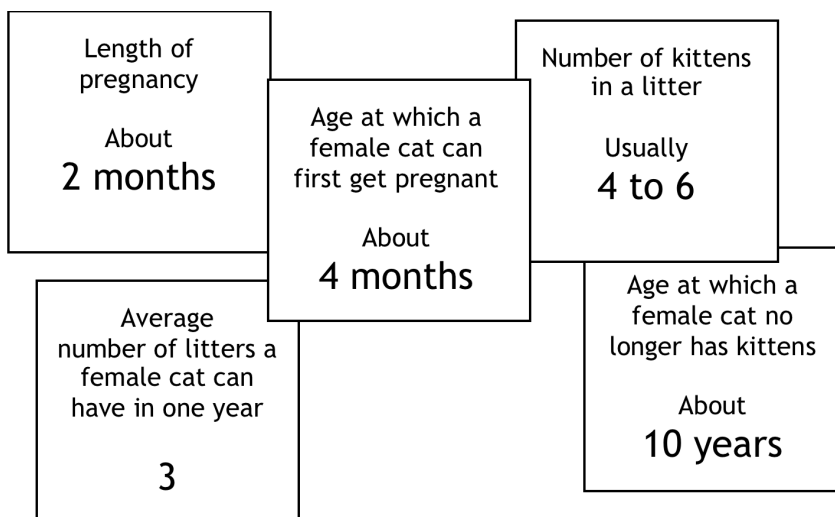
Cats can't add but they do multiply!

In just 18 months, this female cat can have 2000 descendants.

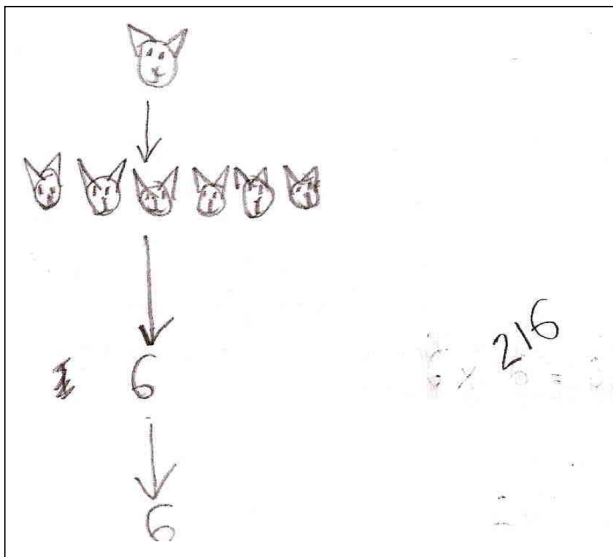
Make sure your cat cannot have kittens.

Work out whether this number of descendants is realistic.

Here are some facts that you will need:

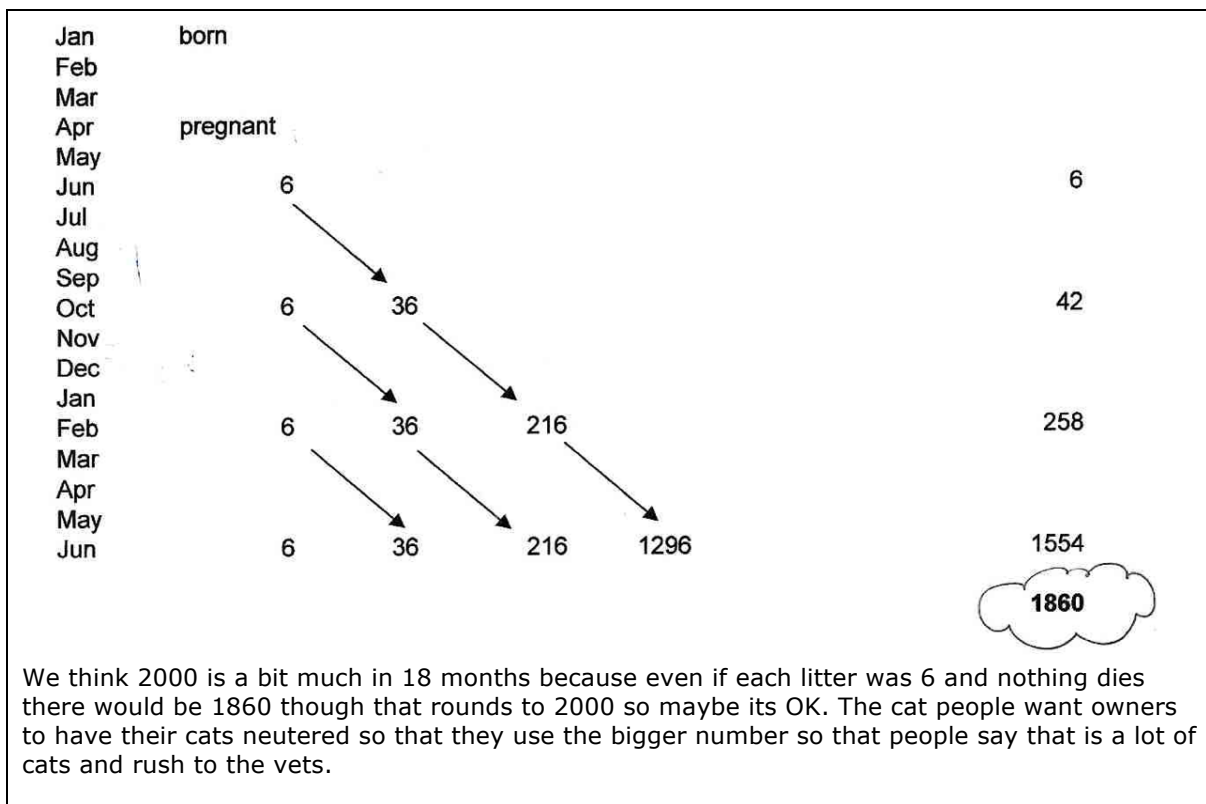


Sample response: Wayne



Sample response: Sally and Janet

Two students worked on this task, discussing and sharing their methods. They used a spreadsheet.



Handout 5: Improving students' responses through questioning

Counting Trees

Sample response: Laura

Laura attempts to estimate the number of old and new trees by multiplying the number along each side of the whole diagram and then halving. She does not account for gaps nor does she realize that there are an unequal number of trees of each kind.

What questions could you ask Laura that would help her improve her response?

Sample response: Jenny

Jenny realizes that sampling is needed, but she multiplies the number of young trees and old trees in the left hand column by the number of trees in the bottom row. She ignores the columns with no trees in the bottom row, so her method underestimates the total number of trees. She does, however, take account of the different numbers of old and new trees.

What questions could you ask Jenny that would help her improve her response?

Sample response: Woody

Woody uses a sample of two columns and counts the number of old and young trees. He then multiplies by 25 (half of 50 columns) to find an estimate of the total number.

What questions could you ask Woody that would help him improve his response?

Sample response: Amber

Amber chooses a representative sample and carries through her work to get a reasonable answer. She correctly uses proportional reasoning. She checks her work as she goes along by counting the gaps in the trees. Her work is clear and easy to follow.

What questions could you ask Amber that would help her improve her response?

Security Camera

Sample response: Max

Max realizes that F and H cannot be seen, but incorrectly thinks that E cannot be seen. He does not show any work to justify his thinking and his further statements are incorrect.

What questions could you ask Max that would help him improve his response?

Sample response: Ellie

Ellie does not show any sightlines to justify her answers. However, she correctly states that F and H cannot be seen and that 3 squares cannot be seen. However, she may be thinking of whole squares rather than areas. Her justification for the 15% is incomplete and poorly explained. She seems to have some understanding that 5% is one twentieth and 10% is one tenth.

What questions could you ask Ellie that would help her improve her response?

Sample response: Simon

Simon correctly states that F and H cannot be seen and that 3 squares = 15% of the area cannot be seen. However, it is possible that he thinks that 3 whole squares are hidden from the camera. He investigates the best place for the camera, and shows that the center of a side is good but he does not investigate further. No calculations are shown.

What questions could you ask Simon that would help him improve his response?

Sample response: Rhianna

Rhianna correctly shows that F and H cannot be seen and that 3 squares = 15% of the area cannot be seen. She investigates the best place for the camera, and shows that the center of a side is good. Rhianna clearly shows diagrams with sightlines and calculations that justify her findings.

What questions could you ask Rhianna that would help her improve her response?

Cats and Kittens

Sample response: Alice

Alice chose to represent the task using a timeline. She has only considered the number of kittens from the original cat. The computation required is accurate.

What questions could you ask Alice that would help her improve her response?

Sample response: Ben

Ben has decided to draw a cat tree', and tries to control for time (with some errors). The communication is reasonably clear, allowing the reader to follow the argument, but the value of 9846 is not explained and does not follow from the reasoning, since, again, only the kittens from the original cat are considered. The number of kittens per litter is made explicit.

What questions could you ask Ben that would help him improve his response?

Sample response: Wayne

Woody appears to favor a minimalist approach! He starts with what would be a time consuming pictorial representation that he then abandons in favor of a numerical representation.

What questions could you ask Wayne that would help him improve his response?

Sample response: Sally and Janet

Sally and Janet used a spreadsheet to control for both time and multiplication and their method is clear and effective.

What questions could you ask Sally and Janet that would help them improve their response?

Handout 6: Suggestions for questions

Phases of problem-solving	Suggested Questions
<i>Related Mathematical Practices</i>	
<p>Formulate questions, choose appropriate representations and techniques.</p> <p><i>MP1, MP4</i></p>	<ul style="list-style-type: none"> • What questions might you ask about this situation? • How can you get started on this problem? • What techniques might be useful here? • What sort of diagram might be helpful? • Can you invent a simple notation for this? • How can you simplify this problem? • What is known and what is unknown? • What assumptions might you make?
<p>Reason logically, construct hypotheses and arguments, compute accurately</p> <p><i>MP1, MP2, MP4, MP6</i></p>	<ul style="list-style-type: none"> • Where have you seen something like this before? • What is fixed here, and what can you change? • What is the same and what is different here? • What would happen if I changed this ... to this...? • Is this approach going anywhere? • What will you do when you get that answer? • This is just a special case of ... what? • Can you form any hypotheses? • Can you think of any counterexamples? • What mistakes have you made? • Can you suggest a different way of doing this? • What conclusions can you make from this data? • How can you check this calculation without doing it all again? • What is a sensible way to record this?
<p>Interpret and evaluate results obtained</p> <p><i>MP1, MP3, MP4, MP6, MP7, MP8</i></p>	<ul style="list-style-type: none"> • How can you best display your data? • Is it better to use this type of chart or that one? Why? • What patterns can you see in this data? • What reasons might there be for these patterns? • Can you give me a convincing argument for that statement? • Do you think that answer is reasonable? Why? • How can you be 100% sure that is true? Convince me! • What do you think of Anne's argument? Why? • Which method might be best to use here? Why?
<p>Communicate and reflect</p> <p><i>MP1, MP3, MP6</i></p>	<ul style="list-style-type: none"> • What method did you use? • What other methods have you considered? • Which of your methods was the best? Why? • Which method was the quickest? • Where have you seen a problem like this before? • What methods did you use last time? Would they have worked here? • What helpful strategies have you learned for next time?

Handout 7: A formative assessment lesson plan

The following suggestions describe one possible approach to a formative assessment lesson on problem solving. Students are given a chance to tackle a problem unaided, to begin with. This gives you a chance to assess their thinking and to identify students that need help. This is followed by a formative lesson in which they collaborate, reflect on their work and try to improve it.

Before the lesson

20 minutes

Before the lesson, perhaps at the end of a previous lesson, ask students to attempt one of the assessment tasks, *Counting Trees*, *Cats and Kittens* or *Security Cameras* on their own. Students may need calculators, pencils, rulers, and squared paper.

The aim is to see how able you are to tackle a problem without my help.

- *You will not be told which bits of math to use.*
- *There are many ways to tackle the problem - you choose.*
- *There may be more than one 'right answer'.*

Don't worry if you cannot understand or do everything because I am planning to teach a lesson on this in the next few days.

Make sure that students are familiar with the context of the problem.

Counting Trees

Does anyone know what a tree plantation is?

How is a plantation different from a natural forest?

The plantation consists of old and new trees.

How might the arrangement of trees in a plantation differ from that of a natural forest?

Cats and Kittens

This is a poster made by a cats' charity, encouraging people to have their cats spayed so they can't have kittens. The activity is about what happens if you don't have your cat spayed and whether the statement on the poster is correct.

Is it realistic that one female cat would produce 2000 descendants in 18 months?

You are given some facts about cats and kittens that will help you decide.

Security Cameras

Have you ever seen a security camera in a shop or on a bus? What did it look like?

Some may not look like cameras at all, but rather like small hemispheres. They may be fixed, but many swivel round. The cameras in this problem can turn right round through 360°.

The drawing shows a plan view of a shop.

This means we are looking down on the shop from above.

The little circles represent people standing in the shop.

Remember to show your working so I can understand what you are doing and why.

Collect in their work and provide constructive, qualitative feedback on it. This should focus on getting students to think and reason - a *Mathematical Practices* agenda. Don't give grades, scores or levels! Write *only* questions below their work. Focus feedback on such issues as:

- *Representing:*
Can you think of a different way of tackling his problem?
What sort of diagram might be helpful?
What assumptions have you made?
- *Reasoning:*
How have you got this result?
Have you checked your calculations?
What would happen if...?
- *Interpreting:*
How can you test the accuracy of your estimate?
What other sample could you have chosen?
- *Communicating:*
I find it difficult to follow your thinking here.
Can you present your reasoning so that someone else can follow every step?

Try to identify particular students who have struggled and who may need support. Also look out for students that have been successful. These may need an extension activity to further challenge them.

Resources needed for the lesson

You will need the following resources:

- One copy of the problem sheet per student.
- Mini whiteboards.
- Large sheets of paper for making posters and felt-tipped pens.
- Calculators and rulers.

Counting Trees

- Spare, large copies of the trees picture for groups to work on together.

Cats and Kittens

- A supply of graph paper or squared paper (if requested.)

Security Camera.

- Spare copies of the plan of the shop for rough working.
- Squared paper (only if requested.)

Re-introduce the problem to the class

5 minutes

Begin the lesson by briefly reintroducing the problem:

Do you remember the problem I asked you to have a go at last time?

I have had a look at your work and I have written some comments at the bottom of it.

Today we are going to work together trying to improve on these initial attempts.

First, on your own, carefully read through the questions I have written on your work. Use your mini-whiteboards to note down answers to these questions.

It is helpful to ask students to write their ideas on a large sheet of paper or mini-whiteboard using felt-tipped pen. This helps you monitor their work and also helps students to share their ideas later in the lesson.

Students work alone responding to your feedback**5 minutes**

Allow the students some time to reflect on your comments and write their responses.

Students work in pairs to improve their solutions**10 minutes**

Ask students to now work in pairs or threes. Give out a sheet of poster-sized paper and a felt-tipped pen to each group.

Now I want you to share your work with a partner.

Take it in turns to explain how you did the task and how you now think it could be improved.

Now I want each pair to work together, comparing their ideas and the feedback I have given. Together, I want you to try to produce an answer to the problem that is better than each of you did separately.

Go round the room, listening, assessing their thinking and making interventions asking strategic questions. Consult a copy of the progression steps for the relevant problem and decide which questions would be most appropriate for moving their thinking towards higher levels of performance. Use strategic questions like:

What is known and what is unknown?

What are you asked to find out?

How can we simplify this problem?

What assumptions have you made?

Students share their approaches with the class**15 minutes**

Ask students to present their ideas and approaches to the class. Focus on their methods rather than their answers. As they respond, use the progression steps to assess their responses. In particular, focus on the quality of the reasoning and communication.

"We decided to count the different types of trees along each side, then multiply these numbers together."

"We drew a time line along the top of the paper and then drew cats underneath to show when they gave birth."

As students present their ideas, ask other students to comment on:

- Representing: Did they choose a good method?
- Analyzing: Is the reasoning correct – are the calculations accurate?
- Interpreting: Are the conclusions sensible?
- Communication: Was the reasoning easy to understand and follow?

Students continue with the problem/ or an extension of the problem **20 minutes**

Encourage students to return to the problem and continue working on it using some of the ideas that have been shared. If they have already produced a good solution, either ask them to find an alternative method, a more convincing reason, or to explore an extension.

Counting Trees

If I now showed you a very large jar of jelly beans how could you estimate the fraction that is red? Write down your method. Can you use what you learned from Counting Trees?

Cats and Kittens

Can you find a simpler, more elegant way of presenting your calculations to Cats and Kittens? Can you use a diagram of some kind?

Security Camera

There are several places that the camera might be placed that are as good as the one you have found. Try to find all the solutions. Can you convince me that these are all possible solutions? Can you explain why they all give the same coverage of the shop?

Collect examples of students' work for the follow-up discussion. Try to assess how much students have learned from the sharing session.

Handout 8: The effects of feedback on students' learning

Read the following two extracts from Black and Wiliam and respond to the questions that follow:

The dangers of giving marks, levels, rewards and rankings

Where the classroom culture focuses on rewards, 'gold stars', grades or place-in-the-class ranking, then pupils look for the ways to obtain the best marks rather than at the needs of their learning which these marks ought to reflect. One reported consequence is that where they have any choice, pupils avoid difficult tasks. They also spend time and energy looking for clues to the 'right answer'. Many are reluctant to ask questions out of fear of failure. Pupils who encounter difficulties and poor results are led to believe that they lack ability, and this belief leads them to attribute their difficulties to a defect in themselves about which they cannot do a great deal. So they 'retire hurt', avoid investing effort in learning which could only lead to disappointment, and try to build up their self-esteem in other ways. Whilst the high-achievers can do well in such a culture, the overall result is to enhance the frequency and the extent of under-achievement.

- What are the implications of this for your practice?
- What would happen if you stopped giving marks or levels on pupils' work?
- Why are so many teachers resistant to making this change?

The advantages of giving clear, specific, content-focused feedback

What is needed is a culture of success, backed by a belief that all can achieve. Formative assessment can be a powerful weapon here if it is communicated in the right way. Whilst it can help all pupils, it gives particularly good results with low achievers where it concentrates on specific problems with their work, and gives them both a clear understanding of what is wrong and achievable targets for putting it right. Pupils can accept and work with such messages, provided that they are not clouded by overtones about ability, competition and comparison with others. In summary, the message can be stated as follows:

Feedback to any pupil should be about the particular qualities of his or her work, with advice on what he or she can do to improve, and should avoid comparisons with other pupils.

- What are the implications of this for your practice?
- Does this kind of feedback necessarily take much longer to give?

Black, P., & Wiliam, D. (1998). *Inside the black box: raising standards through classroom assessment*. London: King's College London School of Education 1998.